On the Small-Time Local Controllability of Cone-Wise Linear Systems

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Abstract

The notion of small-time local controllability(STLC) was first introduced by Sussmann [Sus87]. This property has been extensively studied for linear systems. Although control constraints of conical type have also been considered, the corresponding analysis in the presence of state constraints remains less developed. We investigate the STLC property for piecewise-defined systems on $\mathbb{R}^n$ associated with a polyhedral conical partition of the state space, with a linear control system assigned to each cone. Neighboring cones intersect along switching surfaces on which both corresponding dynamics are admissible. Our approach is based on projecting the original discontinuous dynamics onto a suitably chosen linear subspace, thereby defining a reduced control system. The latter projection is constructed so that trajectories of the reduced system can be lifted to trajectories of the original system by suitable controls. This allows us to characterize the STLC of the original system at the origin in terms of that of the reduced system. We also discuss the case where no nontrivial subspace is available for such a reduction. Our results may be viewed as an extension of earlier work by Veliov and Krastanov in [VK86] and by Krastanov and Quincampoix in [KQ13], who treated cases corresponding to cone partitions generated by intersections of at most two hyperplanes. These results are of local nature and our approach is geometric and constructive. For closely related studies on cone-wise linear systems, check [CHS08; SG11].

References

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